Making Connections: Expanding the Bridging Question Strategy (BQS) to Sorting and Logic Problems

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Abstract

The authors present the second of three follow-up articles to their original article that appeared in the Fall 2007 issue. The Bridging Question Strategy (BQS), as previously presented, is a constructivist strategy for activating prior ordinary concepts and knowledge and connecting them to new concepts and knowledge in mathematics. This article expands on the concept with other examples from sorting activities and logic problems.

A Review of the Bridging of the Bridging Question Strategy

The basic BQS is the process of connecting ordinary knowledge that is not especially content-related to an upcoming content-related topic. The general bridging question asks, “What is going on here?” in reference to some activity or situation. This is an open-ended question, which calls for more than a simple description. BQS begins with knowledge of an everyday activity or situation. It bridges from this to a topic that is related via some easily understood concept or principle. In the original article the BQS used the concept of wrapping and unwrapping a package to bridge to concepts and processes used in solving equations; it also discussed how deciding what new car to buy can be related or bridged to classifying polygons. The previous article presented the process for implementing the BQS. These steps, in abbreviated form, are:

1. Present a bridging activity to bring the concept or topic into focus.

2. Fully activate the bridging question by using group work, using different modes and having students develop their own bridging questions.

3. Bridge to a more remote concept. This can be done by having students predict the relationship between the bridging question and the more remote concept. This may also involve the teacher applying coaching techniques.

An Illustration from Set Concepts: Bridging to Venn Diagrams

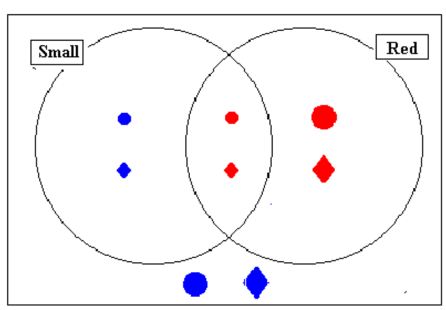
<u>Principle</u>: *Sets of objects or concepts can be illustrated using Venn Diagrams, which assist making deductions about them.*

Some bridging strategies center on a common principle that represents prior knowledge and then bridges to a type of instance that is more remote. However as indicated earlier, a bridging strategy may use any means to relate something that might seem remote to something that is likely to seem more accessible. The concept of a Venn Diagram is a basic concept of logic; a Venn diagram is a visual tool that is used for representing logical relationships. It is composed of overlapping circles, with the overlapping regions representing shared characteristics. The visual mode is very important for helping students create knowledge. Visual representations provided by Venn diagrams enable students to verbalize relationships more effectively. As the following examples illustrate, it can be used with elementary students for sorting activities, and for higher level thinking such as logic problems.

<u>Example</u>: To bridge to this concept, we start with an attribute game and focus on how it can be used with elementary school children. This game can also be

used at a higher level to bridge to additional concepts, such as checking syllogisms. Moreover most adults who have played the game have enjoyed it. Below are pictures of the tokens for a set of the 8 attribute items that are used for one version of this game. Items vary by the attributes {size, color, shape}. Sizes are large and small. Colors are blue and red. Shapes are circle and diamond. A larger set of attribute items can also be used, but even college students can benefit by starting with the simplest version of the game.

This game uses 6 labels for sizes and colors and shapes. It is played on a board with 2 intersecting circles. These circles partition the attribute items into 4 subsets each having 2 elements, depending on the values chosen for the labels. As a prelude to the game, place the labels as in the sample diagram. Ask how the tokens for these items should be placed. Expect some doubts, but you are likely to find some students who will place them as indicated.



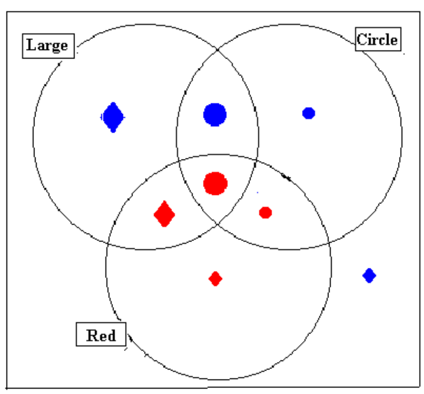
Make sure that everyone can explain why each item can only be placed in this manner. For instance, you might ask questions like the one below.

Question: In a diagram you are told that **LRC** (large red circle) goes in the far right region of the diagram, but you cannot see the labels. Is there another item whose location you can know with certainty? Explain.

Answer: We can deduce that **SBD** (small blue diamond) belongs in the far left region of the Venn diagram as follows: Items inside a region must share an attribute. Since lrc is in the far right region, **SBD** cannot be in that region. Items outside a region must share an attribute. Since lrc is outside of the right circle, **SBD** must be inside of the far left region.

Attribute Game Rules: The game involves 2 teams, with 2 or 4 people on each. To start, Team 1 selects labels of different attribute types and places them face down beside the circles. Team 2 chooses any token. Team 1 must put this token in the correct place. Team 2 must place the remaining tokens in the 4 indicated sets. On each trial, Team 1 either verifies the choice or removes the token if it is in the wrong place and gives it back to Team 2. Team 1 collects one point each time they return a token, however if they return a token incorrectly or allow it to be placed incorrectly, they forfeit the game. Team 2 may try a rejected token elsewhere or try a different token. Once Team 2 has placed all the tokens they must either correctly identify the labels or forfeit the game. The goal of the game for Team 2 is to identify the labels while giving Team 1 as few points as possible in the process. If they correctly identify the label then Team 1 and Team 2 reverse roles and play is repeated. If there is no forfeit then the team with the most points is the winner. The same rules apply to games using 3 circles. The game can also be played with more attribute items.

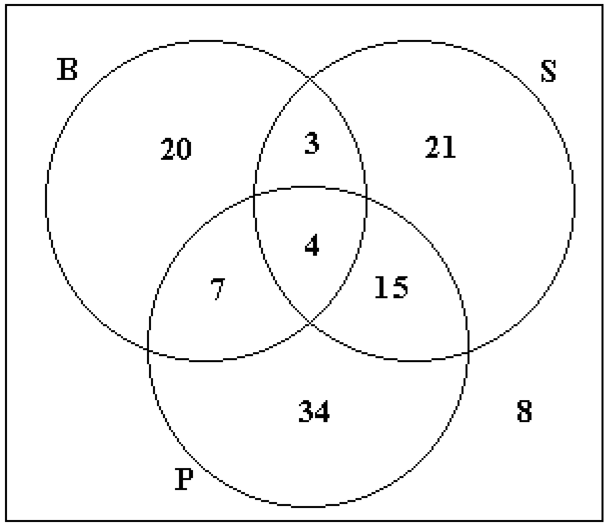
Suggested Use: The purpose of playing and discussing the game is to make the use of the regions seem extremely relevant to the experience of the student. This is also the purpose for either playing or discussing the 3-circle version (below). For elementary children such activities should probably be spread over more than one session. When using the game in a class, we suggest the following. Divide the class into groups of six to the extent possible, as teams of size 3 seem to work best. However opposing teams need not be the same size. Have each group play the game and record any observations, following this by a discussion involving the whole class. Let them play the game at least once more. Next discuss strategies used game playing. For instance, when a token placement is rejected some people try it some other place but others try a different token.



If there is an interest then have them play a game, using 3 circles. Otherwise have the class at least chose labels and place pieces. If you only use attributes of different types then each region will have exactly one token. However you can use a larger set of attribute items and more labels. You can also allow using labels of the same type. Introduce the term 'Venn Diagram' and discuss the general concepts of union and intersection. Illustrate the use of Venn Diagrams with some other situations, such as the vegetable situation below

Vegetable Activity: This situation indicates an arithmetic use of Venn Diagrams. A survey was conducted about the role vegetables in family diets. The most frequently used cooked green vegetables were {Peas, Spinach, and Broccoli}. One question asked which of these the family used on the average at least once a week. The following information was gathered: Broccoli 34, Spinach 43, and Peas 56. Of those, 7 used both Broccoli and Spinach, 11 used both Broccoli and Peas, 19 used both Spinach and Peas. There were 4 families that used all three vegetables. There were 8 families who did not use any of these vegetable at least once a week. How many said only Broccoli? How many said only Spinach? How many said only Peas? How many families were surveyed? How many families used exactly two of theses vegetables at least once a week?

To construct a Venn diagram the number of families for each practice for this information, begin by placing the 4 in the middle region. The notation BS is short for $B \cap S$, i.e. for the intersection of the sets B and S. Since there are 7 in BS place a 3 in the other part of BS. The 7 and 15 can be placed in a similar fashion. Since there are 34 in B, there are 20 in broccoli only. There are 21 in spinach only, 34 in peas only. We can add the numbers in all regions, giving 112 families in the survey. Taking 3 + 7 + 15 gives 25 families who use exactly two of theses vegetables at least once a week.



Other Bridges: The attribute game can be used as a starting point to bridge to some other topics, including some that can be used in college level courses in Boolean Algebra and Informal Reasoning.

Illustrations from Logic:
Bridging to Boolean Satisfaction of Conditionals

Principle: A conditional statement is considered false only when its antecedent is true and its consequent is false.

For a conditional of the form 'If X then Y', 'X' and 'Y' are called the antecedent and consequent respectively. Given a two-valued logic, a conditional is considered false only when its antecedent is true and its consequent is false. In all other cases the conditional is considered true. This requirement for being true involves a perspective that seems artificial or puzzling to many people, and thus they have trouble with the truth table for conditionals in a logic course. Perhaps this is because in ordinary discourse we do not always use conditionals in this two-valued manner, nor do we use them as precisely. However there is another logic concept called Boolean satisfaction that is closely related to our ordinary concept of satisfaction, and there may be less of a problem in bridging from our ordinary uses of conditionals to the Boolean satisfaction criteria below for conditionals. Moreover satisfaction is actually a more basic concept than logical truth, although since it is usually first introduced in first order logic, this is not apparent in the way logic is usually taught. The Boolean version is as follows: ***A conditional fails to be satisfied only when its antecedent is satisfied and its consequent is not satisfied. In all other cases the conditional is satisfied.***

<u>Satisfaction for Ordinary Conditionals</u>: Before turning to Boolean satisfaction in logic we recommend examining how the concept of satisfaction relates to various ordinary uses of conditionals, including several types where it may not seem to apply. This is developed in detail with imaginary discussions in a paper on our website. Here we only sketch the main ideas.

<u>Conditional Rules</u>: A rule says that if you go to Europe then you must obtain a passport. The only way to violate this rule is to go to Europe without a passport. You can satisfy this rule by staying home, regardless of whether or not you obtain a passport. You can also satisfy it by obtaining a passport, regardless of whether or not you go to Europe. In general, for any such rule 'satisfy' is taken in a passive sense. To satisfy a rule is merely to not violate it, and does not involve doing anything else. You only violate a conditional rule when the antecedent applies to you and you violate the consequent. In any other case you have satisfied the rule.

Conditional Claims: Joe's mother claims that if he eats too much candy the he will get sick. Although this is a claim rather than a rule, it works the same way. The only way to check her claim is for him to eat too much of the candy. If he does not get sick her claim was wrong. If he gets sick her claim was correct. Not eating too much is no guarantee that he won't get sick, so whether or not her claim is true, it will be satisfied if he does not eat candy. In a claim like this, one alternative is to refine the satisfaction concept into strong and weak satisfaction. From this perspective, if he does not eat candy there is no way to check the claim, so it neither fails nor is it strongly satisfied. It is only weakly satisfied. In logic, satisfaction means being weakly satisfied. So we do not mean that the claim is correct. We merely mean that nothing has been done to test it. Boolean satisfaction can also be applied to general conditional principles. A general principle differs from a rule in that it is a statement rather than a command. Consider the principle that all crows are black. To state this as a conditional we might say that if it is a crow then it is black.

Deliberately Vacuous Conditionals: A deliberately vacuous conditional is one in which an antecedent considered incorrect is followed by an extravagant consequent that everyone is expected to consider totally implausible. For instance, suppose Bill claims the he can make 50 straight free throws. Jill might say that if you can do this then I can make a thousand. Supposing Bill cannot, Jill's conditional will be vacuously satisfied. Rather than denying his claim, this is her way of scoffing at it.

Biconditionals: One reason that Boolean satisfaction may not seem to apply to some conditional is that conditional language is often used with an implicit intention of stating a biconditional. That is, a person may say ‘If X then Y’ when they mean this as well as ‘If Y then X’. In most ordinary contexts we are able to implicitly recognize when conditional language is to be treated as if it was a biconditional. However this mixed use of conditional language is one reason that some people forget the way conditionals are used logic. For instance, Bob’s mother said told him that if he did his homework then he would be allowed to go to the movies. He did not do the homework, but went to the movies. When she reprimanded him he told her that he did not violate this conditional. It would have been violated only if he had done the homework and she had not allowed him to go. His explanation was unconvincing. She said that he knew that she also meant that if he was going to be allowed to go to the movies then he must do his homework.

Predictions: Conditional used as predictions may not seem on the surface to be best captured by Boolean satisfaction, although we can adopt a perspective that does use Boolean satisfaction. The only time we count a conditional prediction as incorrect is when the antecedent is satisfied but the consequent fails. However we do not usually think of it as otherwise being correct. Even an unlikely prediction such as ‘If the prime rate decreases by 2% next quarter then the Dow Jones average will drop 900 points during that quarter’ cannot be counted as incorrect if the prime rate does not fall by 2%. However we may still tend to think of it as incorrect in principle and logicians have a way to construe this as a specific instance of an incorrect general conditional.

Recommendations: Earlier we recommended that if there is an interest, then have the students play a game using 3 circles. If there is no interest, you can either play the game or not without going against the advice. In this sense you would have at least weakly satisfied the recommendation. The only way to go directly against this recommendation is to observe an interest but not play the game.

Counterfactuals: The most debatable use of Boolean satisfaction may be in relation to counterfactuals, although logicians have a way of accounting for it even there. A counterfactual conditional is one whose antecedent can be considered as possible even though in fact it is false. For instance consider the claim that if Grant had been in command of the Union Army in 1861 then the Civil War would have lasted less than a month. We would not consider this conditional as correct, although it is clear that the antecedent fails.

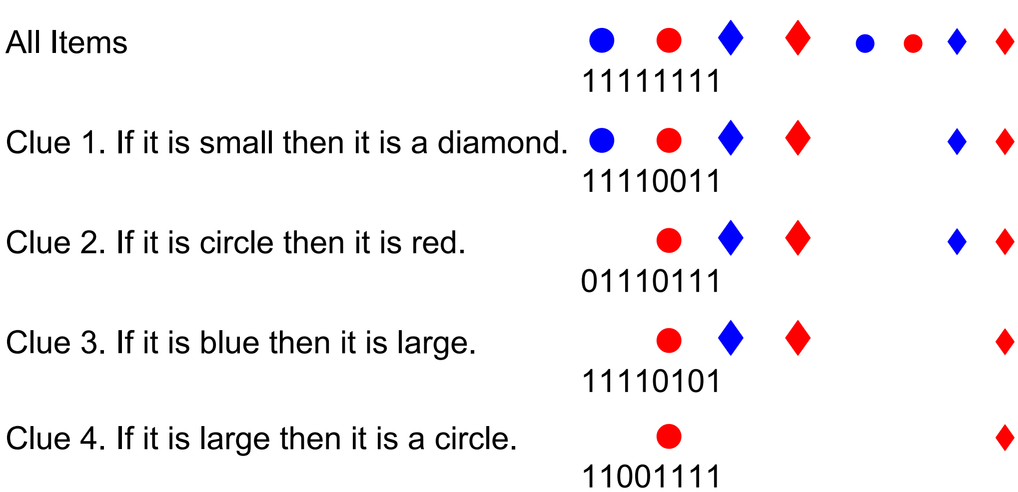
Bridging to Boolean Satisfaction in Attribute Logic

The set of 8 attribute items can be used to bridge to many concepts of logic, including many used in advanced logic courses. *An Approach to Mathematical Logic* by F. Richard Singer III is available without charge on our website. It uses these items, along with a larger set, not only to focus on satisfaction for conditionals but for most other uses of satisfaction in logic. This includes the validity of inference rules and logical deductions. The website includes a large number of attribute puzzles that can be used not only for propositional reasoning but also for reasoning with relations and reasoning with universal and existential quantifiers.

<u>A Simple Sample Puzzle</u>: Suppose that an item was hidden in a box and we had the following clues. If it is small then it is a diamond. If it is circle then it is red. If it is blue then it is large. If it is large then it is a circle. If it is a diamond then it is blue.

We could determine the item by manipulating tokens representing these items, using each clue to eliminate items incompatible with that clue. The remaining items satisfy that clue, at least in the passive sense of satisfaction. For instance, Clue 1 eliminates only the 2 small circles, because we only consider items that satisfy the antecedent and do not satisfy the consequent. We keep the other two items considered along with items not considered.

<u>Solution</u>: Clues could be used in any order. The bit code shows which items a clue eliminates regardless of the order in which it is used. Only the second column has all ones. Solving puzzles using bit codes can be used to bridge to Boolean algebra.



Clue 5. If it is a diamond then it is blue. 
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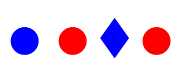
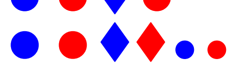
A Further Bridge

A powerful method allows you to deal with many individual items concurrently. The method just used doesn't have that feature. For situations involving a small number of items this is a minor limitation. However if we develop alternate methods for simple puzzles it is often possible to extend the ideas involved to more complex ones. More remote reasoning for solving such puzzles can be illustrated by the deduction below.

(1) S D (2) C R (3) B L (4) L C (5) D B clues
(6) D L by (5) (3)
(7) L by (6) (1), since it cannot be small.
(8) C by (7) (4)
(9) R by (8) (2)

<u>Valid Inferences</u>: In step (6) we indicated that D L was inferred from D B and B L. To say that this is a correct inference means that anything that satisfies D B and B L must also satisfy D L. This may seem obvious, but it may still be instructive to see how it can be checked in a manner that is more accessible to the learner. It should be easy to see that only 4 items satisfy D B and B L and that they all satisfy D L.

Items that satisfies D B and B L  111001
Items that satisfy D L  111101

All other inference can be checked in a similar manner. For instance S D and D L are satisfied by the same items as L. However in a set with medium size items this inference would not be correct. It depends not only on logic, but also on the law for our set of items that can be written as, S L.

Summary

This article examined how the BQS can be used to bridge from an accessible activity to one which may be more abstract or remote. The use of Venn diagrams can be used to bridge from basic, accessible sorting activities to abstract activities such as logic problems.

References

Huetinck, L., & Munshin, S. N. (2004). *Teaching mathematics for the 21st century: Methods and activities for grades 6-12.* Upper Saddle River, NJ: Pearson Merrill Prentice Hall.

Pelech, J., & Singer, R. (2007). The bridging question strategy. *Journal for the Practical Application of Constructivist Theory in Education,* 2(1).

