"Can I teach like that?" Guiding Construction of Geometric Knowledge in a Third Grade Classroom

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Abstract

In response to the growing awareness that the constructivist theory of learning can inform effective educational practices (Fraivillig, 1999), university-based educators worked with a practicing teacher to design and test a series of geometry lessons for a third-grade classroom to explore how teachers might plan for and implement an approach to learning based on student construction of personally meaningful understandings. In this article we discuss the philosophy of constructivism and how it can be applied to mathematics instruction in geometry. We share the reaction of the students and classroom teacher, who were accustomed to more traditional teacher-centered methods of instruction.

What is constructivism?

When planning a study of the process of learning from a constructivist perspective, a paradox is encountered. According to its fundamental principles, constructivism maintains that understanding and knowledge are constructed in the mind of the individual learner. Yet, when doing research, those carrying out the study must have a common understanding of the study's elements, and must be able to form hypotheses about how they believe the participants will create their own knowledge. The key to resolving the paradox that two researchers may not have the same understanding about constructivism is that knowledge can be "taken-as-shared" – that understandings among individuals can "fit" even if they

do not “match,” allowing us to form common understandings and to speculate how others might develop knowledge (Cobb, 2000). In order to explore classroom learning from a constructivist perspective, we first developed a shared understanding of the meaning of constructivism.

There are an assortment of definitions of constructivism in the literature, but they all agree it is fundamentally a philosophy or of learning (Clements, 1997) and can be viewed as a theory of knowing (Glasersfeld, 1987). One of its principles is that an individual’s knowledge can only be generated from experience, not from merely being introduced to information (Simon, 1997). Thompson (2000) concurs, adding that constructivism orients us to matters of what people know and how they come to know it. Ernest (1995) ties this to the Piagetian concept of adaptation, explaining that constructivism involves the building of mental structures through restructuring of established mental structures.

We used these descriptions as our working definition of constructivism – a philosophy of learning that is consistent with the belief that students construct their own knowledge through adaptation of experiences into knowledge structures.

How does constructivism influence pedagogy?

Constructivism is not a pedagogical method (Clements, 1997) and does not endorse any particular pedagogical approach (Thompson, 2000). However, as a

philosophy it influences the way teaching and learning take place by informing and improving pedagogical practice (Ernest, 1995).

Constructivists do not believe students construct learning by happenstance (Clements, 1997; Fraivillig, 1999). They believe teachers must develop plans to structure and guide student exploration without attempting to lead them to a particular response (Simon, 1995).

Clements (1997) describes how a constructivist framework is manifested in the classroom. Although it is commonly thought that active or cooperative learning techniques are required, he explains traditional lectures can also be used. The vital component is student reflection on and sense-making of the lesson content, while actively constructing relationships with prior understandings. Questions posed by the teacher can stimulate cognitive conflict, encouraging students to extend their knowledge and build deeper understandings (Cobb, 1994). Without prior understanding, students cannot assimilate corresponding new knowledge (Glasersfeld, 1997). Teachers must be sensitive to individual subject-matter experience and current knowledge structures of students.

They should also be aware of how the group of students exerts influence on learning among themselves through their interactions (Thompson, 2000). As each learner constructs personal meaning, the teacher and students can interact to negotiate socially accepted knowledge (Ernest, 1995). Ernest emphasizes

that social construction of knowledge should take place through discussion, collaboration, and negotiation. With these understandings of the constructivist perspective of pedagogy, we approached the task of guiding student learning in geometry.

Purpose

For this project, we chose to plan and demonstrate lessons designed to reflect a constructivist framework. One of the authors had previously observed the third grade where we would teach for a college course. Her observations revealed that the teacher favored direct instruction math lessons with the teacher as the distiller of knowledge. As "distiller of knowledge" the teacher, feels she holds all the correct answers during a lesson. Therefore, in addition to teaching within a constructivist framework we hoped to provide professional development for the teacher.

Although there is not one right way to teach, mathematics reforms based on research provide a variety of effective teaching strategies that encourage children's understanding of mathematics, their ability to solve problems and their disposition towards mathematics (National Council of Teachers of Mathematics (NCTM), 2000). We wanted to demonstrate some of these techniques to this teacher as a means of having her reflect on her methods of teaching. We wanted to emphasize the use of discussions as a means of developing mathematical concepts and the use of manipulatives for problem solving.

The setting

The third grade classroom we worked in was a pre-K to 8th grade school in a northeastern city. It was a SURR (School Under Registration Review) school. (The Commissioner of Education places those schools that are determined to be the farthest from State standards and most in need of improvement under No Child Left Behind to be named Schools Under Registration Review. They receive additional resources, monitoring, and a timeline for improvement.) The classroom consisted of twenty-four students. There were more males than females, and all but one student were African American. The teacher, Mrs. Smith, is European American. An African American aide works in the room for part of the day.

The children sat in assigned seats in rows of three. An aisle separated three rows on the left from three rows on the right. A reading rug area was in the back of the room as well as a small table. The children sat at their seats for lessons for the majority of the day. The math lesson usually occurred for 50 minutes in the afternoon.

Our observations

Initially we visited the classroom to observe Mrs. Smith teaching math to her class. The lessons we observed were didactic in nature. Mrs. Smith led the children through the math lesson using questioning. The purpose of the questioning was to find the “correct answer” rather than to guide the students’

understandings as defined by national mathematical standards (NCTM, 2000). There was an expectation in the classroom that students would always raise hands to answer, not talk to their neighbor and face front to pay attention to the teacher.

During one observation, Mrs. Smith worked on solving problems. The school had adopted a version of a K-W-L chart as a school-wide math problem-solving tool. K-W-L charts can be used to develop discourse and shared understandings since they record what is known about a subject, what the class or student wants to know, and eventually, what was learned. This day, the class was working on a worksheet of word problems. Mrs. Smith directed the students as they worked on the problems.

One of the problems on the worksheet was:

> *Suppose you are planning a picnic for 34 people. You must buy paper plates in packages of 8. How many packages of paper plates will you need?*

The dialogue occurred as follows. (Unless noted, children raised their hands and were called on.)

> Mrs. Smith (pointing to the first column of her K-W-L chart, "Know"): "What do we know in this problem?"
> N: (Calls out. Mrs. Smith reminds her to raise her hand.) Count by 8.
> Mrs. Smith: "We're not looking for the answer. What do we know?" (No answer). "Should we keep counting 8, 8, 8, 8?"
> D: "I don't understand."
> Mrs. Smith: "Pretend you and Dad are going on a picnic..."
> C: "We need 5."
> Mrs. Smith: "Don't give me the answer. Tell me what we know."
> C: "Count by 5's? …10's?"
> N: "Count by 2's?"

Mrs. Smith wanted to walk step-by-step through the problem. After the problem was read, C immediately used his manipulatives to set up a method for solving the problem. He was told to not solve the problem but “stay with the class.” Mrs. Smith is not interested in the answer at this point in the lesson although some children are ready to solve it. She was focused on steering the children through the problem using the K-W-L strategy. This appeared to confuse the students; they were used to finding the answer and here the expectations changed. They did not understand why this change occurred. As the lesson progressed, Mrs. Smith directed the students’ use of counters (representing the picnic paper plates in the problem).

This scenario was typical of the math lessons we observed. We noticed several consistent behaviors. The teacher did most of the talking. The discourse in the class consisted of children talking to the teacher with permission, giving her an answer or telling her they did not understand. Only once did Mrs. Smith ask a child to explain their answer. Discussions of mathematic thinking did not take place during the lessons we observed.

Although lecturing and guided instruction can be used to help children construct their mathematical knowledge, reflection and sense making (Clements, 1997) are the vital components of a lesson. Student engagement with the information and

ideas is essential for developing understanding. In this classroom, the teacher's behavior did not encourage reflection but rather procedural competence.

Manipulatives were used in the math class to follow the teacher's step-by-step solution to the strategy. Using manipulatives in an imposed manner often makes teachers feel that their students are "actively learning" (Clements, 1997). However, the manipulatives were not being used in a manner consistent with the insights of constructivism. The children were not allowed to construct their own solutions with the manipulatives, but instead only allowed to mimic the teacher's model. After the problem was read, one student had immediately used his manipulatives to set up a method for solving the problem. He was told to not solve the problem, but to "stay with the class."

Our lesson planning

Mrs. Smith asked us to teach the first week of a unit called 'Using Geometry." This unit included the concepts of congruency, flips, turns and slides, intersecting and parallel lines, lines of symmetry and solids. Reviewing the expected outcomes for this unit we found that the children needed to create a design using three shapes, with congruency, and intersecting and parallel lines. The understanding of congruency, intersecting, and parallel lines became the learning goals for our planning.

We thought about activities that would help students develop the unit's understandings. We did not know the students and our previous observations revealed little about their knowledge of these learning goals. For an understanding of the students' prior knowledge of the concepts, we relied on mathematical teaching guides and informal questioning of second and third grade acquaintances including educators and children.

Our working definition of constructivism caused us to reject the textbook activities that relied on introducing terms and identifying examples on worksheets. Based on our experiences as teachers, we thought that in order to reach the learning goals the children needed to actively manipulate objects to gain understanding and to reflect upon the schemas they were developing.

Mrs. Smith relied on direct instruction, so we wanted to demonstrate how discussion could encourage the construction of mathematical concepts. Because several of the concepts (lines of symmetry, congruency, and intersecting and parallel lines) were definitions, it seemed important to build a shared understanding of these ideas. In our planning, we decided upon activities that would allow interactions among the students. It was our hope that interactions would help everyone understand that others have different perspectives (Ernest, 1995), and begin to foster the construction of shared knowledge (Cobb, Wood & Yackel, 1993). We wanted to connect the concepts to other mathematical ideas and to student experiences outside of mathematics lessons (NCTM, 2000).

The teaching experience

The first author taught five math lessons over a two week period. The lessons were 50 minutes long. In this section we will describe the first day's activities in some detail to provide the reader with a sense of the classroom activity. After that we will briefly describe particular activities to illustrate additional points.

Our first goal was to develop the students' knowledge of the concept of congruency. When asked about the term, most of the class seemed to have an understanding of this idea. Geometrically, items are congruent if they are the same size and shape. But, a "congruency desk hunt" revealed differences in understanding of "same."

After we held up an item, the children were asked to search for a congruent item in their desk. The first item was a glue stick. Several children pulled one out of their desk and the following conversation occurred:

R (researcher): "Can you tell me how these are congruent?"
A: "They've the same circles on the bottom."
B: "No, they're different colors."
R: "You're right, they are different colors. But are they the same size?"
(The boy looks unsure. R holds her glue stick next to the one the boy thinks is different.)
B: "Yes, same size."
C: "They're the same shape, too."
D: "They have the same words, 'Glue Stick.'"

The researchers realized the children were concentrating on numerous attributes not just size and shape. The next item for the hunt was scissors, which the researchers thought would reduce the number of attributes the children focused

on. A few children pulled out scissors. The researcher asked if they were congruent.

T: "No, yours are blue (his were purple)."
R: "What does someone else think?"
C: "They're the same size."
J: "Not the same shape."

J's comment caused others to shake, "No," in disagreement. The researchers thought the scissors were congruent and asked J to show how they were different. J pointed to the finger openings. One set had finger holes that were round but the other pair had more elongated holes. J was the only one who noticed this. Her understanding of "different" proved to be more accurate. Drawing the holes on the board helped the students to see what J meant.

Without these conversations, our view of the children's understanding would have been different. If J had not explained her reasoning the researcher might have assumed J was confused. Developing an atmosphere where students' explanations are respected provides opportunities for teachers continually assess students' understandings. Interactions provide an opportunity for cognitive conflict that will encourage students to reflect on what their peers say. It is worth noting that the children in this classroom did not hesitate to explain their answers because this was not the typical interaction in the lessons we observed.

Another activity this first day was to use geoboards to create 3 and 4-sided figures. The researchers challenged the students to create several different shapes which allowed discussions about which students had made congruent

shapes. The children recorded their congruent shapes on geoboard paper and labeled them. During the discussion, it was difficult for the researchers to assess who fully understood the concept and who was merely repeating justification mentioned earlier in the lesson. The shapes the students recorded provided a means to assess the student understanding, in this discussion-based, cooperative activity.

After each lesson, we discussed what worked and what did not. We also reflected on what concepts we thought the children were developing an understanding of and modified our learning activities accordingly. Ernest (1995) suggests that responding to children's inputs and reactions is one of ways teaching from a constructivist perspective is unique. Responding and reacting happened as the activities occurred but also in our reflecting afterwards. This process was shared with Mrs. Smith.

The second day's activities were designed to introduce and develop notions of flips, turns and slides. We rejected the textbook activities; they used worksheets and did not have the students label the movements. Discussing the goals prior to the lesson, we agreed upon activities that involves moving cut outs of polar bears while using the terms flip, turn and slide, and discussing the meaning with the students. While teaching, we became concerned that these were new concepts to the students and we were trying to develop them in isolation. Again we had

children record some work and again we adjusted our next teaching activities based on our observations and the children's work.

As we discussed modifying the next day's lessons, we debated whether we were using too many different tools in our activities. Each day we were using a manipulative and a recording task. We were concerned that Mrs. Smith would reject this type of teaching because of the need to prepare materials. She earlier commented about not using learning centers because of the amount of work. Our concern that we model lessons that were less work to prepare led to a more structured lesson the last day we taught. We wanted to demonstrate that questioning could lead children to reflect and make sense of the concepts even within a more structured environment. This day's activities focused on reviewing some of the concepts already presented. Unlike previous days, the researchers set the pace for the activity by asking the children to show their understanding based on a specific, individually-completed task. As each child indicated they were finished, questions were used to engage the child in explaining his or her thoughts. As a final assessment, we asked the children to illustrate and label several terms such as parallel, congruent, etc. During this process, we hoped the teacher would reflect on our use of conversation and questioning when attempting to help the children see other viewpoints and articulate their understanding of the concepts.

The teacher's perspective

Mrs. Smith, the classroom teacher, was interviewed twice. The first interview took place after the classroom geometry activities were completed, in order to get her immediate reaction to the experience. The second was conducted four weeks later, after she had time to assess her students on topics included in the geometry lessons and to reflect on the overall effect of the experience on her teaching and on the children's learning.

Mrs. Smith is a middle-aged woman who has taught elementary school for almost fifteen years, seven of them as a third grade teacher at the school where she is currently teaching. When she was approached about allowing us to teach her students, she cautioned us that they were a particularly poorly behaved group so we might have some trouble managing them.

The first interview – immediately following the teaching episodes

At the beginning of the interview we reviewed the concept of constructivism with Mrs. Smith, explaining to her that it is a philosophy of learning that espouses the belief that learning occurs through children adapting their experiences to create their own understanding and knowledge. We further explained that, according to this philosophy, it was unlikely that students would learn by merely being told information, and that in our geometry lessons we were trying to explore learning from this constructivist perspective. Mrs. Smith remarked that she felt the

philosophy made sense. She briefly discussed the importance of being aware of what her children already know when planning what to teach them.

We asked Mrs. Smith to remark about the method of teaching she observed being used during the geometry lessons. She said she enjoyed watching the class during the lessons and that she thought children should be exposed to a variety of ways of teaching. She said she thought our methods of introducing new concepts included some useful ideas and that they worked well with her students. She seemed especially pleased that her students had cooperated with us and had worked together without disagreements occurring and without students getting off-task in spite of the less structured environment.. She believed the students gained a thorough knowledge of the topics through the classroom activities and discussions, even without using the paper/pencil lessons they typically did. She said that it was as if they were “using all different parts of their brain,” allowing them to learn in a broader, deeper way.

Mrs. Smith expressed concern that doing this type of classroom activity herself would be difficult because of the amount of planning time and preparation of materials involved. She had asked for our materials when we finished each lesson, and in the interview expressed hope that she would, at least, be able to teach those geometry lessons in the future using our method because she could use those materials again.

She also thought the teaching methods we used made it difficult to monitor class-wide learning. The approach she typically used to make sure everyone was learning was to poll the class or have them reply to questions in unison. She felt our less structured learning environment would require her to move among the students listening to their discussions and monitoring their work, in order to spot patterns of error. She could then correct them by addressing the class as a group. However, she felt this was a less efficient method than her own.

The second interview – four weeks after the teaching episodes

The second time we spoke to Mrs. Smith, we again asked whether she thought she would use more constructivist-based methods in the future in her classroom. She seemed more willing to consider the possibility at this interview. She expressed hope that, over time, she could gradually change her method of teaching to increasingly use constructivist-related methods because they had "worked so well." Mrs. Smith thought they would be beneficial not only in mathematics, but in all subjects. However, she felt it would take many years for her to revise all of her teaching to use these methods because of the amount of work involved in planning new lessons and creating new materials.

We asked how the students had done on her assessment of the topics they had explored while we were in the classroom. She said they had done remarkably well on the test and that she had not done any additional teaching or even review of those topics.

She remarked that she believed the high level of student engagement during learning had improved their recall on the test. She said the lessons had helped students to “build memories;” allowing them to “think back on it [the lesson] and remember” what they had learned because the learning experience was so memorable. She felt this was a result of the nature of the lessons and that the experience was a unique one for them.

Our reactions

We felt we had demonstrated a more constructivist approach to teaching mathematic concepts. By allowing and encouraging interactions, the children were led to reflect on the activity and concept, and on others’ answers. The students were enthusiastic about sharing their reasoning. This was a change from our observations where Mrs. Smith seemed to question only for the correct answer.

Another goal of our teaching was for the students to enjoy mathematics. NCTM (2000) states that knowing and communicating about mathematics can be personally satisfying. By creating enthusiasm for mathematical activities, we do not assume children will increase their mathematical knowledge. Yet, we hoped having some fun may encourage them to be open-minded about mathematics and allow learning and understanding to occur. We feel that our teaching did this; the children were participating in the activities and sharing their thoughts.

We were not trying to force Mrs. Smith to change her teaching style but only to demonstrate other ways mathematics learning could occur. Hopefully through this experiment, she will see the value in considering other approaches to teaching, including the positive effect it can have on learning.

It is important to note that Mrs. Smith's classroom plans, and thus our activities, were constrained by the need for students to perform successfully on assessments because they attended a SURR school and the assessment results were closely monitored. We were asked to provide activities that would lead to the students being able to answer specific test items correctly. Unfortunately, the nature of some of the test items required us to focus classroom dialogue directly on how that type of question might be answered.

Conclusion

In the current educational climate, where standardized assessment to prove overall student progress is required, educators must find methods that both teach children effectively and lead to successful performance on such. With sufficient reflection on lesson preparation, it is possible, and perhaps even easier, for teachers to use constructivist based methods to accomplish this. As we learned from this experience, engaging students in stimulating activities and dialogue results in both deep learning and good recall.

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