The Quality Secondary Math Classroom

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Abstract

The author expands on the recommendations of the Mathematical Sciences Education Board, as outlined in its 2001 pamphlet, "What Should I look for in a Math Classroom." Strategies for developing and creating the recommendations of the MSEB are described in detail, along with specific examples. The article makes the connection between constructivist theory and its applications in secondary mathematics classrooms.

Introduction

The goal of this analysis is to be able to recognize a quality math classroom, both from the observer's point of view and from the teacher's point of view. The 2001 MSEB pamphlet, "What Should I look for in a Math Classroom?" (see Appendix) suggests how to reach that goal, but it is missing the "How" aspect. This paper breaks the pamphlet into common themes and attempts to answer the "How." In other words, "How do I, the teacher, meet these requirements and what would a good lesson look like?"

Basic Considerations

As an overview of high quality math teaching on the secondary level, the following points apply in all cases. For each lesson, a teacher should start with an attention getter. For example, a **bell-ringer** question or problem of the day could be used to start the class. This opener gets the students focused on mathematics, and it can be used for reviewing prior

knowledge as well. Next the teacher should state the objective(s) of the new lesson and present **new material** through an anticipatory set. Research has shown that students learn best within the first 20 minutes of a 45-minute class (Sousa, 2001), so that is the perfect time for the students to learn something new. When teaching new material a teacher should consider what knowledge the students would be able to construct on their own. For example in a probability unit, before teaching any theoretical probability, put students into small groups and have them conduct a set of experiments that allow students to discover concepts and relationships on their own. Bring everyone back together for whole group discussion and explain to the students the theoretical probability. Students often will be able to offer the formulas, tree diagrams, sample space and so on, so you end up being a coach, not a talking head.

After new material is presented and practiced, old material, **homework**, and student questions should be addressed. The class should conclude with a **closure** activity that would tie all the learning together. Journal writing is an excellent way for a student to attach sense and meaning to the new learning, thus increasing the probability that it will be retained.

Examples of each part of a learning episode are included in the vignettes that follow. One other important point would be for the teacher to assign only 1 or 2 new problems for homework, and 6 - 8 problems from the topic taught the previous day, plus a few problems from a week ago. This strategy of spiraling the homework allows the student to bring previously learned information into working memory where it can be transferred into long-term storage. (Sousa, 2001) In short, the latest brain research on cognition and learning has important implications for the way homework is assigned!

In the following sections, the information from “What Should I look for in a Math Classroom?” as been grouped according to related topics. (For reference, the original pamphlet from the MSEB is printed in the Appendix to this article.) The recommendations are then illustrated broken with actual activities that might be seen in a typical teaching episode in a secondary math classroom.

What is happening in the classroom?

From the MSEB pamphlet:

1) Multiple resources (each of these resources has the common goal of helping students to gain understanding and increase retention.)

    a) Students are using textbooks as only one of many resources. Manipulatives such as blocks and scales and technology such as calculators and computers are useful tools, and students should be learning how and when to use them.

    b) Teachers are bringing a variety of resources into the classroom from guest speakers to creative use of technology.

    c) Teachers are using manipulatives and technology when it is appropriate, not just as “busy work.”

Lessons need to be structured so that every student, from the visual learner to the tactile-kinesthetic learner, can be reached. No matter what the learning style of the individual, each student will benefit from seeing and hearing a well-designed explanation of new concepts and then manipulating objects that help demonstrate the concept. Careful scrutiny of the curriculum will lead to the conclusion that there are plenty of places where manipulatives and technology can be used. Some examples of manipulatives include blocks, cards, tiles, dice, spinners, flashcards, and scales. Some good applications of technology include Geometer’s

Sketchpad, graphing calculators, PowerPoint presentations, motion detectors, and internet activities like Scavenger Hunt and WebQuests.

The following vignettes demonstrate what a good math classroom that uses multiple resources might look like. There are examples of bell-ringers; activities for acquiring and integrating new material, for extending and refining that material, and for using knowledge meaningfully (Marzano, 1992); and how to close a lesson as well as use manipulatives and technology.

> *Mr. Dee is teaching a unit on relationships inside a circle. The topic for the day is the measure of central and inscribed angles and their arcs. After the students have finished their bell-ringer for the day, Mr. Dee gets them into the computer room to work on Geometer Sketchpad. He hands them an instruction sheet that clearly outlines the steps to open Geometer Sketchpad, start a new sketch, and create a circle. He then has them build central angles and calculate the number of degrees in the angle and the number of degrees around the arc of the angle. Students do this repeatedly for several central angles. Jeremy states that the measures are equivalent every time and the rest of the class nods in agreement. Mr. Dee then has the student make inscribed angles and measure the angle and then the arc. Students discover that the angle is one half of the arc. July poses the question: "Mr. Dee, is it always true that the measure of the central angle is equivalent to Its arc and the measure of an inscribed angle is one half its arc?" Mr. Dee replies: "Do each of you have the exact same circle?" Class: "No, they are all different." "Did each of you have the same results as far as the measure of the central angle and its arc?" When all of the students reply affirmatively, they have developed their own conclusion, and Mr. Dee confirms their discovery.*

> *Ms. Eff's topic of the day is relations and functions; during the lesson she taught the similarities and differences between circles, ellipses, parabolas and hyperbolas. At the end of her explanation and notes, she has each student make up a relation of their own and hand it to another student. She then has all the parabolas go to corner #1, all the hyperbolas go to corner #2, and so forth, and explain to each other why they are in that corner. Then she asks any relation that crosses the x axis only go to corner #1, y axis only go to corner #2, both the x and y axis corner #3 and explain to the others why they went there. Then she asks any relation that has a center of 0,0 go to corner #1 and any relation with no center point go to corner #2. She divides them up over and over again based on similarities and differences in their relation equation, then she has them switch with another student and she repeats the corners. In this way, Ms. Eff assesses whether the students know the differences in relations by how they interact with each other in the corners.*

Ms. Gee just finished teaching a fabulous lesson about relations and functions. She quickly goes over homework and then assigns a journal question. "Explain how you know that $25x^2 + 16y^2 = 36$ is an ellipse, not a circle. Describe this ellipse. Then explain how we could change it into a hyperbola, and describe the hyperbola." The students write out their response to the journal question and hand it in before leaving. Journal writing is a very effective strategy to solidify new information and promote retention. Ms. Gee reads all of the responses and discovers that the lesson she taught was indeed truly fabulous.

*Mrs. Bee found a wonderful interactive website: (*http://www.math.ucla.edu/~ronmiech/Actuarial_Review/Related_Rates/Master/Master.html) *She instructs her calculus class that the website has problems that demonstrate related rates. Each of the students must go to the Website and successfully complete each one of the related rate problems. (Solutions are outlined on the Website; Mrs. Bee does not mind because she believes that beating yourself up over a solution never helped anyone learn.)*

From the MSEB pamphlet:

2) Real life math counts

   a) Students are becoming aware of how math is applied to real life problems, not just learning a series of isolated skills. And as in real life, complex problems are not solved quickly.

   b) Teachers are working with other teachers to make connections between disciplines to show how math is a part of every other major subject.

   c) Teachers are raising questions that encourage students to explore several solutions and challenge deeper thinking about real problems. They are not just lecturing.

   d) Teachers are encouraging students to go on the next challenge once a step is learned, understanding that not all students learn at the same pace.

   e) Teachers are exploring with students career opportunities that emphasize mathematical concepts and applications.

When students are presented with problems that are challenging, they enjoy working hard to find the answers. The following vignettes demonstrate what a good math classroom that applies math to real life might look like.

Ms. Eye decided to let her students discover how logarithmic and exponential functions apply to human beings. She handed each of her students a growth chart for children of age 2 to 18. The students then pick either height or weight and pick a percentile from the growth chart. She told them that they were to assume the role of a computer programmer working for a doctor's office and their assignment was to create a formula that would represent their chosen height or weight and the percentile to use in a program. The goal was to incorporate all of the different formulas into one program so that the doctor could easily see what percentile a patient was in based on the patient's height and weight.

Mrs. Bee has decided to start off her graphing unit with a practical experiment. She has the whole class stand in a circle and clasp hands. Billy steps aside with a stop-watch and Jesse stands at the board next to a T chart labeled "number of students| number of seconds." When Billy says go, Mrs. Bee starts the wave. It travels from her to the student she is holding hands with on the right. Then the wave moves around the circle, when it gets back to Mrs. Bee she shouts "Stop" and Billy stops the stopwatch. Jesse records the number of students in the wave and the number of seconds it took to complete one wave. One student sits down and they do the wave again. They do the wave removing one student each time for 6 more trials. Jesse records the data each time. When they are finished, Mrs. Bee asks the student to make a scatter plot of the data. Mrs. Bee then poses the questions: " What would the time have been if only three people were in the wave? What would the number of people have been if the wave took 85 seconds?" From this activity the students see a need for creating a line of best fit.

Ms. Que decided to take advantage of her classroom being on the second floor. She had in her possession enough motion detectors for her students to team up. She asked the students to find the speed of a falling object. Each team decided what they wanted to drop, how high they wanted to drop it from, and how they would use the motion detector to collect the data. Then they dropped their object, collected and organized their data, and with very little teacher involvement, they presented to the class the speed of their object. Discussion as to why there were differences in speed given different objects and heights ensued. The Physics teacher came in and added information about gravity in a vacuum, the affects of friction, and terminal velocity.

From the MSEB pamphlet:

3) There is more than one way to skin a cat

   a) Students are realizing that many problems have more that just one "right" answer. Students can explain the different ways they reach a variety of solutions and why they make one choice over another.

   b) Teachers are allowing students to raise original questions about math for which there is no "answer in the book," and promoting discussion of these questions, recognizing that it may be other students who will find reasonable answers.

Research supports initiatives such as constructivism. Students need to know that math is not a spectator sport; they must actively participate by rolling up their sleeves and getting their pencils moving. Many problems will require them to think hard and put new concepts together. In most cases, if the student understands the concepts, memorizing a formula becomes unnecessary because they construct the essential tools when needed. This is why understanding the process for solving a particular type of problem is emphasized over memorizing formulas.

The following vignettes demonstrate what a good math classroom that teaches students to look at problems in multiple ways might look like.

> *Ms. Jay starts off every class with a problem up on the board. Today's problem for her Math A class is: "A truck leaves from point A to point B traveling 40 mph. The truck arrives at point B in one hour. When the truck is half way to point B, a car leaves point A traveling 50 mph. How far has the car traveled when the truck reaches point B?" The students are accustomed to the schedule and enter the classroom with the expectation of immediately getting to work on the problem of the day. Each day there is a different type of problem to do. These problems force the students to remember what they learned earlier in the year. Today, while the students focus on the bell-ringer, Ms. Jay gets homework passed back, attendance taken, and excuses out of the way. After 5 minutes, Ms. Jay asks the students what their solution was. George stands and says: "25 miles." Ms Jay then asks George to go up to the board and show the class the work he did to get the answer. John says; "I got the right answer but I did it differently." Ms. Jay has John show how he attained his correct answer. I*

discussion ensues about all the different ways there are to get a correct answer.

There were three days until Christmas break and Mr. El just finished a statistics unit in a high school math class. As a culminating activity he decided to show the movie "The Grinch Who Stole Christmas." Before showing the movie he broke the class into three groups; the "Christmas" group, the "Grinch" group and the "Who" group. He explained to the students that while they watched the movie they needed to keep track of how many times the word "Christmas", "Grinch" or "Who" is said in each five-minute interval. Mr. El shows them an example of a bar graph that he has made as well as a scatter plot for similar data. He discusses the meaning of mean, median and mode and how they relate to his data. Each group is then tasked with reporting their interpretation of the data from the movie using math vocabulary and solid reasoning as well as displaying their data in a creative and meaningful manner.

From the MSEB pamphlet:

4) We are all in this together

a) Students are working in groups to test solutions to problems. They are more than only "listeners" and are highly involved.

b) Students are Interacting with each other, as well as working independently, just as adults do at work.

c) Students are working in a physical setting that promotes teamwork and helps them challenge and defend possible solutions. Even while using computers, they do not always work alone but with other students, helping each other.

d) Students are learning how to communicate mathematical ideas with one another.

e) Teachers are moving around the room to keep everyone engaged and on track. They are not glues to the chalkboard.

f) Teachers are drawing on student discovery and creativity to keep them interested. The teacher knows that boredom is the enemy of learning.

The following vignettes demonstrate what a quality math classroom that promotes group-work might look like. There are examples of bell-ringers, acquiring and integrating new material, closures as well as the use of manipulatives and technology.

After teaching a particularly difficult topic, Mr. Zee decided to secretively see if the students have understood the material. He had each of the students get up, go and get a dry erase slate, marker and eraser and sit back down with their group of three. He then assigned each group a number and each group member a letter. So each group had an A, B and C person. He put on the chalkboard a problem from the day's lesson. Each student attempted the problem on their slate, and then discussed their solutions with their group. After a minute of small group discussion Mr. Zee asked each of the A's to hold up their slates. He checked each group's A's answers and awarded a point to each correct team. He immediately knew which students understood the lesson and which ones needed assistance. He asked another question. Each student tried the problem and then discussed it in their groups. Mr. Zee heard various students teaching the process to other members of their group. He heard "Oh, I get it" and "now I can do it" from students that had not understood. He was able to assess their progress and correct any misunderstandings all in an activity the students thought was a game.

As a review for an upcoming logic exam Ms. Dubya made 7 decks of review cards. On one card she had a sentence and on another card she would have that sentence in symbols. On one card she would have a truth table missing one value and on another card she had the missing value. Each deck had a total of 55 different matching pairs. Her students were arranged into groups of three and given a deck of cards. Ms. Dubya said go, and each group started to match the cards together. They talked about their choices and worked continuously until each card had a partner. The first group finished, with all of the pairs matched correctly received bonus points. After finishing one game each group shuffled the cards and traded with another group to try again.

Mr. Kay wants to make sure his students understand how to solve one-step equations. He muses; "What better way to check to see if they understand than to have them create a one-step equation of their own?" He decides to create a worksheet that he calls "Send a Problem." Mr. Kay hands each student the worksheet that has three sections, one for the problem, one section for the solution, and the last section for the check. He asks each of his students to write an equation on scrap paper. Each student is then asked to write a word problem that would fit his or her equation. They have to send their word problem to another student who should write the equation that fits the word problem and solve the problem. That student then hands the problem to another student who must check the work already done and then check the

answer. The problem is then handed back to the originator who lets the others know if they were correct of not.

From the MSEB pamphlet:

5) Test what you can do, not just what you have memorized

a) Teachers are using assessments that reflect the way math is being taught, stressing understanding and problem-solving skills, not just memory.

Monday Mr. Kay started a unit concentrating on Metric Conversions. Friday he decided to have the students fill out a self-assessment. The feedback from the students helped Mr. Kay assess himself and adjust the following week's lessons to best meet the needs of the students.

Student Self Assessment

Name:_____

!. Name 2 most important things you have learned this week in Math class.

2. Name at least 1 sort of problem with which you still need help.

3. How do you feel at this moment about Math class? (circle all that apply.)

interested confused successful bored happy relaxed worried clever rushed frustrated any words of your own:_____

4. How could we (that means all of us) improve Math class?

The vignette above describes a covert assessment. The use of journals is a quick and easy way to see if the students have understood the lessons each and every day. The use of slates in the form of a game allows the teacher to assess understanding. The culminating activities are wonderful tools to see if the students can apply what they have learned to real world situations. Student self-assessments help both the students and the teacher

understand what information the students understand and what information they may need more practice on. Tests and quizzes can be designed so that students are applying their knowledge and explaining their solutions using appropriate mathematical language, not just demonstrating processes and finding answers.

IV. Conclusion

It is hoped that the examples provided in this article help to extend the MSEB recommendations to demonstrate *how* they can be applied. This can give a much more complete picture of what one should *really* look for in a quality math classroom.

Appendix

What Should I Look For in a Math Classroom?

(Mathematical Sciences Education Board, 2001)

A math classroom should provide practical experience in mathematical skills that are a bridge to the real world of jobs and adult responsibilities. This means going beyond memorization into a world of reasoning and problem solving.

Sounds good, but how will I recognize a good math classroom when I see it?

Look for these changes from the traditional classroom, and if you see them, you will be looking at a classroom that is preparing students for the world outside of school.

What are students doing?

- Interacting with each other, as well as working independently, just as adults do at work.
- Using textbooks as only one of many resources. Manipulatives such as blocks and scales and technology such as calculators and computers are useful tools, and students should be learning how and when to use them.
- Becoming aware of how math is applied to real life problems, not just learning a series of isolated skills. And as in real life, complex problems are not solved quickly.
- Realizing that many problems have more than just one "right" answer. Students can explain the different ways they reach a variety of solutions and why they make one choice over another.
- Working in groups to test solutions to problems. They are more than only "listeners" and are highly involved.
- Learning how to communicate mathematical ideas with one another.
- Working in a physical setting that promotes teamwork and helps them challenge and defend possible solutions. Even while using computers, they do not always work alone but with other students, helping each other.

What are teachers doing?

- Raising questions that encourage students to explore several solutions and challenge deeper thinking about real problems. They are not just lecturing.
- Moving around the room to keep everyone engaged and on track. They are not glued to the chalkboard.

- Allowing students to raise original questions about math for which there is no "answer in the book," and promoting discussion of these questions, recognizing that it may be other students who will find reasonable answers.
- Using manipulatives and technology when it is appropriate, not just as "busy work."
- Drawing on student discovery and creativity to keep them interested. The teacher knows that boredom is the enemy of learning.
- Encouraging students to go on to the next challenge once a step is learned, understanding that not all students learn at the same pace.
- Bringing a variety of resources into the classroom from guest speakers to creative use of technology.
- Working with other teachers to make connections between disciplines to show how math is a part of every other major subject.
- Using assessments that reflect the way math is being taught, stressing understanding and problem-solving skills, not just memory.
- Exploring with students career opportunities that emphasize mathematical concepts and applications.

LEARNING also takes place outside of school. Thinking mathematically is critical to every life skill from balancing a checkbook to understanding the newspaper. In every job people use math skills that require the ability to identify a problem, look for information that will help solve the problem, consider a variety of solutions and communicate the best possible solution to others.

LOOK CLOSELY AT A MATH CLASSROOM IN TODAY'S SCHOOLS:

Is it teaching the same old stuff in the same old way, turning out students grossly unprepared for the real adult world?

OR

Is it teaching skills for life and work, which do we choose for our children. ...you look and decide.

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